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Report on the PhD thesis of Ms. Ewa Maksymiuk

This thesis of Ms. Ewa Maksymiuk, entitled *Exact solutions of the relativistic Boltzmann equation in the relaxation time approximation*, was prepared under common supervision of prof. dr hab. Wojciech Florkowski and dr hab. Radoslaw Ryblewski. The thesis is based on 5 papers published from peer-reviewed international journals (4 from Physical Review C and 1 from Journal of Physics G).

The quark-gluon plasma (QGP) created at RHIC turns out to go like a liquid with low viscosity. One of the central questions in relativistic heavy-ion collisions is what its properties are. In the hydrodynamics evolution, dissipative processes are characterized by the transport properties. Viscous hydrodynamics is used to describe the space-time evolution of the created matter in heavy-ion collisions via the Boltzmann kinetic equation. The main subject of this thesis is to solve the Boltzmann equation for a mixture of quarks and gluons in the relaxation time approximation and to compare closely with the predictions of anisotropic hydrodynamics which recently attracts more and more attention.

Section 1 is devoted to a brief introduction to the hydrodynamic description of ultra-relativistic heavy-ion collisions. Here the author defines clearly the questions to answer. In subsection 1.5, a short summary of the main results is given, and this is helpful to keep track of the physics behind the concrete results to be presented subsequently.

Section 2 is devoted to a pedagogical introduction to the kinetic equations in the relaxation time approximation. A set of moments of the coupled kinetic equations are introduced and they will play the important role in ensuring conservation of the baryon number and energy-momentum.

Section 3 is devoted to the construction of the formal solutions of the kinetic equations under the baryon number and energy-momentum conservation with a given set of anisotropic distribution functions for quarks and gluons. A brief derivation of the relevant quantities is outlined and their explicit forms are summarized in Appendices A-D.

Section 4 is devoted to the concrete numerical results with 3 sets of initial conditions reflecting different anisotropy of the system. In subsection 4.2, the proper-time dependence of the effective temperature and chemical potential is presented for a small and a large values of the initial baryon number density. A comprehensive study with classical and quantum statistics is also performed. In subsection 4.3, the same quantities are compared with the results from the Navier-Stokes (NS) hydrodynamics as well as those from the ideal hydrodynamics. It is shown that the non-equilibrium dynamics switches over rather fast to the NS hydrodynamic regime. This is clearly seen also in the shear viscosity. One interesting observation is that the hydrodynamization in the shear sector sets in earlier than that in the bulk sector, which is strongly supported by the work in the context of the gauge/gravity duality [129]. In subsection 4.4, the ratio, the longitudinal to the transverse pressure, is considered. Because of the technical simplification made in this study, the ratio obeys a rather simple scaling of the proper time. It is shown that the ratio depends only weakly on the mass of the constituents and the type of statistics.

Section 5 is devoted to how the coupled kinetic equations are solved under the strict conservation of the baryon number and the energy-momentum. Accordingly, 7 equations are deduced analytically.

Section 6 is devoted to the numerical results of the 7 coupled equations. The kinetic approach with the exact solutions constructed in the earlier section is compared with aHydro. It turns out that they agree fairly well.

Section 7 is devoted to a short summary of this thesis.

The thesis is closed with fruitful appendices and a rich bibliography.

There arise the following questions:

1. As the author honestly admits in the abstract and in the main text, whole study has been carried out under 2 major assumptions; (i) a common relaxation time to quark and gluons and (ii) the relaxation time being constant everywhere. Its verification or possible update leaving those assumptions out will certainly require further effort. Does the author have any intuition that such simplification would still capture the right physics?

2. On page 14, it is written that the author uses the estimates of the transport coefficients from the Israel-Stewart (IS) approach that fixes the issues of instability and acausality in the NS. In Sections 4 and 6, the results are compared with the NS approach. Has this been done with the IS actually?

3. Figure 4.7: What is the origin of the negative bulk viscosity before the onset of hydrodynamization?

4. It is perhaps an error, but Eq. (4.7) misses the contribution from the gluons.

5. I am afraid of a misinterpretation of the expression for the bulk viscosity given in Ref. [125]. We have derived the general expression for both bosons and fermions. Therefore, in case of a mixture of them, we implicitly meant that the derivatives κ_1 and κ_2 must be evaluated with the total pressure of this mixture. In our subsequent paper, Nucl. Phys. A **832**, 62 (2010), we calculated the shear and bulk viscosities in a system composed only of dressed fermions.

6. I do not get the physics intuition on the quantities A_1 and A_2 presented in the last part of Section 4. What is the origin of the appearance of the 2 attractors? How do we understand the different convergence in A_1 from that in A_2?

7. To me the summary is rather incomplete. In Section 1.5, 5 main results are listed. In Section 7, however, the last point on the above list is not mentioned. Also, I miss a prospect after the study summarized in this thesis.

Minor comments:

1. There is a typo on page 10, in sec.1.1.1: "wQCD" should be "wQGP".
2. There is a typo on page 34, right after Eq. (3.33): "the total longitudinal momenta" should be "the total longitudinal pressure".

Conclusions

The thesis of Ms. Maksymiuk contains important new results concerning the microscopic description of anisotropic hydrodynamics within the kinetic approach. Special emphasis was put on the conservation of the baryon number and the energy-momentum, which is sometimes disregarded, to solve the Boltzmann equation. The exact solutions for the 7 major anisotropic quantities were constructed in the relaxation time approximation. The thesis is written to a large extent in a self-contained fashion and can be used as a nice introductory material for students who want to join this field.

I conclude that the thesis clearly complies with all the requirements. I would like to warmly recommend this thesis to the Institute of Physics of Jan Kochanowski Univeristy for defense.

C. Sasaki

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